



COURSE DESCRIPTION CARD - SYLLABUS

Course name

Introduction to approximation theory [S1MwT1>E-WdTA]

Course

Field of study

Mathematics in Technology

Year/Semester

4/7

Area of study (specialization)

–

Profile of study

general academic

Level of study

first-cycle

Course offered in

Polish

Form of study

full-time

Requirements

elective

Number of hours

Lecture

30

Laboratory classes

0

Other (e.g. online)

0

Tutorials

15

Projects/seminars

0

Number of credit points

4,00

Coordinators

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Lecturers

Prerequisites

Basic knowledge of mathematical analysis and differential equations. Using of basic notions and methods of mathematical analysis and differential equations. Student has a ability to think logically. Student is aware of the need to expand their competences. He understands the need for learning.

Course objective

Familiarizing students with various aspects of approximation theory and its applications, and general problems of this division of mathematics.

Course-related learning outcomes

Knowledge:

- student knows the object of approximation theory and basic types of problems;
- student knows basic approximation theorems in spaces of continuous functions;
- he knows basic methods of approximation of functions by algebraic polynomials and trigonometric polynomials and their applications.

Skills:

- student can use knowledge of higher mathematics;
- student is able to determine convergence or divergence of sequences and series (Advanced Level); he solves problems using estimation;
- student is able to solve problems using basic approximation methods and approximation theorems.

Social competences:

- student is aware of the level of their knowledge in relation to research in exact and technical sciences;
- student is aware of the deepening and expansion of knowledge to solve newly created technical problems.

Methods for verifying learning outcomes and assessment criteria

Learning outcomes presented above are verified as follows:

Learning outcomes presented above are verified as follows:

Lectures:

assessment of knowledge and skills demonstrated in the written test. The test is assessed in a point system. The condition of passing the test is obtaining at least 50% of points.

Tutorials: two written assignments carried out under the teacher's supervision. The condition of receiving a positive grade from tutorials is obtaining at least 50% of points. Additional points can be obtained for activity during classes.

Programme content

1. Continued fractions and approximation
2. Remarks on convergence of series
3. Selected applications of power series
4. Basic problems in approximation theory of functions
5. Classical Weierstrass's theorems and generalizations of Weierstrass's theorem
6. Korovkin type approximation theorems
7. Approximation of functions by algebraic polynomials
8. Approximation of functions by trigonometric polynomials

Course topics

Lectures:

1. Continued fractions and approximation
 - Definition of finite and infinite continued fraction, reducers and value of continued fraction
 - Approximations of real numbers using continued fraction reducers
2. Some tests for convergence of series
 - Raabe's test, Bertrand's test, Kummer's test, Gauss's test
3. Approximating sums of series
4. Selected applications of power series
 - Solving ordinary differential equations using the power series expansion method
 - Approximation of the solution of a first-order ordinary differential equation using a partial sum of a power series
 - Non-elemental integrals
5. Basic problems in approximation theory of functions
6. Classical Weierstrass's theorems and generalizations of Weierstrass's theorem
7. Korovkin type approximation theorems
8. Bernstein polynomials
 - Bernstein polynomials, basic properties, approximation order and Woronowska type theorems
 - Bézier curves (definition and basic properties, application)
9. Chebyshev polynomials
 - Chebyshev's theorem
 - Chebyshev polynomials and their properties
 - Chebyshev economization
10. Chebyshev series
11. Fourier series
 - Expanding the function into a trigonometric Fourier series in any interval (also in complex form)
 - Uniform convergence and properties of series coefficients Fourier

- Dirichlet kernel, n-th Lebesgue constant
- Selected applications of trigonometric Fourier series

Tutorials:

1. Continued fractions and approximation, some tests for convergence of series
 - Reducers and value of continued fraction
 - Approximations of real numbers using continued fraction reducers
2. Some tests for convergence of series
 - Raabe's test, Bertrand's test, Kummer's test, Gauss's test
3. Approximating sums of series
4. Selected applications of power series
 - Solving ordinary differential equations using the power series expansion method
 - Approximation of the solution of a first-order ordinary differential equation using a partial sum of a power series
 - Non-elemental integrals
5. Basic problems in approximation theory of functions
6. Chebyshev polynomials
 - Abel transformation
 - Chebyshev polynomials and their properties
 - Chebyshev economization
7. Chebyshev series
8. Fourier series
 - Expanding the function into a trigonometric Fourier series in any interval (also in complex form)
 - Selected applications of trigonometric Fourier series

Teaching methods

Lectures:

multimedia presentation accompanied with examples presented on the blackboard and with questions to the group of students;

Tutorials: solving problems on the board, initiating discussion about the solutions.

Bibliography

Basic

- R. A. DeVore, G. G. Lorentz, Constructive Approximation, Springer -Verlag, Berlin 2006.
- G. M. Fichtenholz, Rachunek różniczkowy i całkowy, PWN, Warszawa 2017.
- W. Narkiewicz, Teoria liczb, Wydawnictwo Naukowe PWN, Warszawa 200.
- S. Łojasiewicz, Wstęp do teorii funkcji rzeczywistych, PWN, Warszawa 1972.
- J. Musielak, Wstęp do analizy funkcjonalnej, PWN, Warszawa 1989.

Additional

- W. Pleśniak, Wykłady z teorii aproksymacji, Wydawnictwo UJ, Kraków 2000.
- E. W. Cheney, Introduction to Approximation Theory, AMS Chelsea Publishing, Providence, Rhode Island 2000.
- W. Ł. Daniłow, A. N. Iwanowa, J. K. Isakowa, L. A. Lusternik, G. S. Salechow, A. N. Chowanski, L. J. Cłaf, A. R. Jampolski, Funkcje, granice, szeregi, ułamki łańcuchowe, PWN, Warszawa 1970.
- K. Knopp, Szeregi nieskończone, PWN, Warszawa 1956.
- R. Taberski, Aproksymacja funkcji wielomianami trygonometrycznymi, Wydawnictwo Naukowe UAM, Poznań 1979.
- J. Niedoba, W. Niedoba, Równania różniczkowe zwyczajne i cząstkowe, AGH, Uczelniane Wydawnictwo Naukowo-Dydaktyczne, Kraków 2001.

Breakdown of average student's workload

	Hours	ECTS
Total workload	100	4,00
Classes requiring direct contact with the teacher	45	2,00
Student's own work (literature studies, preparation for laboratory classes/ tutorials, preparation for tests/exam, project preparation)	55	2,00